5. (HL[1], ex. 3.2-3, page. 79) Suppose that you have decided to invest *6 000 m.u.*. Two different friends, Peter and Joan, have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planed by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Become a full partner of Peter's venture would require an investment of *5 000 m.u.* and *400 hours*, and your estimated profit would be *4 500 m.u.*. The corresponding figures for the Joan's venture are *4 000 m.u.* and *500 hours*, with an estimated profit to you of *4 500 m.u.*. However, both friends are flexible and would allow you to come in at any fraction of a full partnership you would like. If you chose a fraction of a full partnership, all the above figures given for a full partnership would be multiplied by this same fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

a) Use the graphical method to identify the feasible region.

b) Compute the objective function value at every extreme feasible point, and identify the optimal solution.

c) In the graphic identify all the possibilities of sequential feasible solutions found by the simplex algorithm.

d) For each feasible extreme point identify the corresponding feasible basic solution (compute all the values for the slack variables). For each basic feasible solution identify the non-basic variables.

e) Solve the problem by the simplex algorithm.

6. A firm has a factory with a weekly capacity of *70 hours*, where 3 three different products can be produced (**P1**, **P2** e **P3**). To produce one unit of **P3** one hour of capacity is enough, however the unit production of **P1** and **P2** needs are, the double and the triple, respectively. The products when ready are stored in a warehouse with *100* m^3 of space available. Each unit of a product needs *1* m^3 to be stored. The unit returns are *10*, *15* e *5 m. u.*, respectively.

a) Formulate the LP problem to maximise the total return.

b) Determine by the Simplex method the optimal BFS.

c) Write a small report explaining the relevant issues of the production plan that should be undertaken.

7. Use the Simplex algorithm to solve exercises **3.a)**, **3.c)**, **3.d)**, **3.e)**, **3.h)** and **3.j)**.

[1] Hillier, Lieberman, "Introduction to Operations Research", 8th edition, McGraw-Hill, 2005.

8. Consider the following problem **P**:

$$Max\ z = x_1 - 3x_2$$

$$\text{s.a:} \begin{cases} \frac{1}{3}x_1 + x_2 \le 8 \\ x_1 - x_2 \le 8 \\ x_1 \ge 0 \end{cases}$$

- **a)** Use the graphical method to solve **P**.
- **b)** Find and classify a solution with x_2 equal to zero and that turns the first constraint into a binding one.
- **c)** Write the standard and the augmented forms of **P**.
- **d)** Apply the first iteration of the Simplex algorithm to find a feasible solution to **P**.

9. Solve the following linear programming problems

a) $Max\ Z = 3x_1 + 4x_2 + 2x_3$

s. to:

$$\begin{cases} x_1 + x_2 + x_3 \le 20 \\ x_1 + 2x_2 + x_3 \le 30 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

b) $Max\ Z = 2x_1 + 2x_2 + 3x_3$

s. to:

$$\begin{cases} 2x_1 + x_2 + 2x_3 \le 4 \\ x_1 + x_2 + x_3 \le 3 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

c) $Min\ Z = -2x_1 + x_2 - x_3$

s. to:

$$\begin{cases} 3x_1 + x_2 + x_3 \le 60 \\ x_1 - x_2 + 2x_3 \le 10 \\ x_1 + x_2 - x_3 \le 20 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

d) $Max\ Z = x_1 + 2x_2 + 4x_3$

s. to:

$$\begin{cases} 3x_1 + 2x_2 + 5x_3 \le 10 \\ x_1 + 4x_2 + x_3 \le 8 \\ 2x_1 + x_3 \le 7 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

10. Consider the Simplex tableaux of a maximization problem:

Basic variables	coefficients of						RHS
	z	x_1	x_2	x_3	x_4	x_5	
z	1	$\boldsymbol{c}$	0	2	0	0	9
x_2	0	-1	1	$\boldsymbol{a_1}$	0	0	3
x_4	0	$\boldsymbol{a_2}$	0	-3	1	0	1
x_5	0	$\boldsymbol{a_3}$	0	4	0	1	2

Find the set of values for a_1, a_2, a_3 and c for which the following statements are true:

- **a)** the current solution is optimal;
- **b)** the current solution is optimal and there is at least one more basic optimal ;
- **c)** the objective function is unbounded.